

Neutrino mass and the Standard Model

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Abstract

It is pointed out (not for the first time) that the minimal Standard Model, without additional gauge-singlet right-handed neutrinos or isotriplet Higgs fields, allows for nonvanishing neutrino masses and mixing. The required interaction term is nonrenormalizable and violates the global $B - L$ symmetry. The ultimate explanation of this interaction term may or may not rely on grand unification.

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I. INTRODUCTION

It is sometimes said that the Standard Model by itself does not allow for nonvanishing neutrino masses. Taking the Standard Model to refer solely to its particle content and gauge-interaction structure [1–10], this is not so. It is, in fact, possible to consider an interaction term in the Lagrange density, which uses only the standard-model multiplets and generates Majorana masses for the neutrinos. This term is nonrenormalizable and does not conserve $B - L$, the difference of the baryon quantum number B and the lepton quantum number L .

Having a nonrenormalizable theory makes sense as long as the Standard Model is not considered to be the definite and final theory. From this point of view, the interaction term discussed here will have crossed the mind of anyone who has pondered the origin of neutrino mass. Indeed, one of the earliest papers to mention this term appeared more than 30 years ago [11]. Still, it may be useful to clarify the basic logic of this term and to emphasize its simplicity.

II. INTERACTION TERM

The Standard Model of elementary particle physics combines the chiral $SU(2) \times U(1)$ gauge theory of the electroweak interactions [1–4] having anomaly cancellations between the different Weyl fermions present [5, 6] and the vectorlike $SU(3)$ gauge theory of the strong interactions [7–10]; further references can be found in, e.g., Ref. [12]. The particle content of the minimal Standard Model consists of the $SU(3) \times SU(2) \times U(1)$ gauge bosons, $N_{\text{fam}} \times 15 = 45$ left-handed Weyl fermions for family number $N_{\text{fam}} = 3$, and a single complex isodoublet scalar Higgs field. In the following, we focus on the leptonic sector (charged leptons l_f^\pm and neutrinos ν_f , with family label $f = e, \mu, \tau$) and use the notation of Ref. [12] in terms of four-component Dirac spinors.

The $SU(2) \times U(1)$ irreducible representations of the lepton families and the Higgs field are of the type $(\mathbf{2})_Y$ and $(\mathbf{1})_Y$, that is, isodoublet and isosinglet with $U(1)$ hypercharge Y . Given the definition of the electric charge $Q \equiv I_3 + Y/2$, the basic Weyl (anti-)fermion fields of the first lepton family (label $f = e$) and the Higgs field are:

$$L_e = \begin{pmatrix} \nu_{e,L} \\ e_L^- \end{pmatrix}_{-1}, \quad R_e = (e_R^+)_{+2}, \quad (1a)$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}_{+1}, \quad \tilde{\Phi} \equiv i\tau_2 \cdot \Phi^* \equiv \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix} \cdot \Phi^*, \quad (1b)$$

where the asterisk in the last definition of (1b) denotes complex conjugation and the three matrices τ_a are the standard 2×2 Pauli matrices for isospin (denoted σ_a for spin). The isodoublet in (1a) has lepton number $L = +1$ and the corresponding isosinglet $L = -1$. The leptons of the second and third families are contained in similar representations, L_f and R_f

for label $f = \mu, \tau$. The usual Higgs vacuum-expectation-value constant v is obtained from $\langle \Phi^\dagger \cdot \Phi \rangle \equiv v^2/2$.

The generalized theory, now, is defined by the local Lagrange density \mathcal{L}_{SM} of the minimal Standard Model [12, 13], to which is added a local lepton-Higgs interaction term \mathcal{L}_5 ,

$$\mathcal{L}(x) = \mathcal{L}_{\text{SM}}(x) + \mathcal{L}_5(x). \quad (2a)$$

Specifically, take the following contact-interaction term which is both $SU(2) \times U(1)$ gauge invariant and Lorentz invariant ($\hbar = c = 1$):

$$\mathcal{L}_5(x) = \frac{1}{M_5} \sum_{f,f'} \left[\lambda_{f,f'} \left(\bar{L}_f(x) \cdot \tilde{\Phi}(x) \right) \left(\tilde{\Phi}^\dagger(x) \cdot L_{f'}(x) \right)^c + \text{H.c.} \right], \quad (2b)$$

where the charge conjugate of the Dirac spinor field $\psi(x)$ is denoted $\psi^c(x) \equiv \mathcal{C} \gamma^0 \psi^*(x)$, with $(\mathcal{C} \gamma^0) (\gamma^\mu)^* (\mathcal{C} \gamma^0)^{-1} = -\gamma^\mu$. The composite field operator on the right-hand side of (2b) has mass dimension five, hence the suffix ‘5.’ The dimension-5 term (2b) has already been considered in Ref. [11].

Expanding the Higgs isodoublet in (2b) around its vacuum expectation value $(0, v/\sqrt{2})^T$ gives

$$\mathcal{L}_5 = \frac{v^2}{2 M_5} \left[\sum_{f,f'} \lambda_{f,f'} \hat{\nu}_f^T (-i\sigma_2) \hat{\nu}_{f'} + \text{H.c.} \right] + \dots, \quad (3)$$

where the superscript ‘T’ stands for transposition and $\hat{\nu}_f$ is the left-handed two-component Weyl spinor corresponding to the four-component Dirac spinor $\nu_{f,L}$ in the chiral representation of the Dirac gamma matrices, $\gamma^5 \equiv i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \text{diag}(1, 1, -1, -1)$. The first term on the right-hand side of (3) contains a mix of Majorana mass terms.¹ *A priori*, there is no connection between the neutrino masses from (3) and the charged-lepton masses.

The ellipsis in (3) contains interaction terms involving the components of the Higgs isodoublet field. In unitary gauge, $\Phi(x) = (0, h(x) + v/\sqrt{2})^T$ with $h(x) \in \mathbb{R}$, the Feynman rules of the new cubic ($h \nu \nu$) and quartic ($h h \nu \nu$) vertices are obtained from the following Lagrange density:

$$\mathcal{L}_5^{(\text{unitary gauge})} = \frac{1}{M_5} \left(\frac{v^2}{2} + \sqrt{2} v h + h^2 \right) \left[\sum_{f,f'} \lambda_{f,f'} \hat{\nu}_f^T (-i\sigma_2) \hat{\nu}_{f'} + \text{H.c.} \right]. \quad (4)$$

If nonzero neutrino masses are taken as input ($\lambda_{f,f'} \neq 0$), these new scalar-neutrino interactions are an unavoidable consequence of our approach and contribute, for example, to

¹ The manifest $SU(2)$ gauge invariance of (2b) and rotation invariance of (3) rely on identical mathematics: for isospin, the identity $\Omega^\dagger \cdot (i\tau_2) \cdot \Omega^* = i\tau_2$ with an arbitrary matrix $\Omega = \omega_a i\tau_a + \omega_4 \mathbb{1} \in SU(2)$ having real parameters ω_μ on the unit 4-sphere [$\sum_a (\omega_a)^2 + (\omega_4)^2 = 1$] and, for spin, the same identity but now in terms of σ_a . The transposition and commutation properties of the Pauli matrices σ_a also make for the manifest invariance of (3) under Lorentz boosts. Remark, finally, that the $U(1)$ gauge invariance of (2b) holds separately for the terms in large parentheses.

flavor-changing neutrino-neutrino scattering $\nu_e \nu_e \rightarrow \nu_\mu \nu_\mu$ at small but finite center-of-mass energies, $0 < \sqrt{s} \ll M_5$. Naive estimates suggest that these new contributions satisfy the supernova bounds [14] on neutrino-neutrino scattering cross-sections but definitive statements have to wait for the proper UV completion of (2), as will be discussed in the next section.

III. DISCUSSION

The interaction term (2b) is nonrenormalizable because of the coupling constant $1/M_5$. This mass scale M_5 may be related to the energy scale at which the $B - L$ global symmetry is broken ($B + L$ is already broken dynamically at the electroweak scale [15–17]). The experimental data from particle physics and cosmology suggest a sub-eV neutrino mass scale [18], which, with $v \sim 10^2$ GeV and $\lambda_{f,f'} \sim 1$ in (3), implies $M_5 \gtrsim 10^{13}$ GeV. But M_5 could also drop to the TeV scale if, for some reason, the couplings $\lambda_{f,f'}$ were of order 10^{-10} .

From a purely theoretical point of view, the neutrino mass scale v^2/M_5 in (3) traces back to the gauge invariance of (2b) [two Higgs isodoublets for the “saturation” of the two lepton isodoublets giving the factor v^2] and nonrenormalizability [giving the factor $1/M_5$]. The same type of mass scale v^2/M_R follows, of course, from the see-saw mechanism [19–23] (brief reviews can be found in Refs. [12, 18]). The see-saw mechanism, in its simplest form, introduces N_{fam} right-handed neutrinos [possibly coming from an $SO(10)$ grand unified theory] and has, per family, an effective 2×2 neutrino-mass matrix with diagonal entries 0 and M_R and off-diagonal entries v (giving eigenvalues M_R and $-v^2/M_R$ for $v^2 \ll M_R^2$). But, here, there are no right-handed neutrino fields and there is no such $2N_{\text{fam}} \times 2N_{\text{fam}}$ matrix to diagonalize, only the $N_{\text{fam}} \times N_{\text{fam}}$ matrix from (3) with entries individually of order v^2/M_5 .

In the context of renormalizable theories, there are also alternatives to heavy right-handed neutrinos; see, in particular, the discussion of Ref. [24]. These different realizations can be expected to lead to different results for the neutrino-neutrino scattering cross-sections discussed in the last paragraph of Sec. II. (For neutrino-neutrino scattering, certain statements in Ref. [24] as to the indistinguishability of the different realizations presumably hold only in the strict low-energy limit, $\sqrt{s}/M_5 \rightarrow 0$.)

Let us make two final comments. First, it is remarkable that all experimental facts of elementary particle physics known to date [18] can be described precisely by the fermion and Higgs multiplets of the minimal Standard Model if one allows for a single nonrenormalizable contact-interaction term in the action. These experimental facts include those from neutrino oscillations and perhaps those from neutrino-less double-beta decay. In principle, even Lorentz-violating effects could be incorporated.²

² A hypothetical superluminal neutrino velocity (first claimed to have been discovered by OPERA [25], but later retracted [26]) could also be described by using only the multiplets of the minimal Standard Model. The idea would be to appeal to spontaneous breaking of Lorentz invariance [27–29] and to take a Majorana-mass-type term as (2b) with derivatives inserted between the two gauge-invariant terms in

Second, the origin of the term (2b) may very well rely on an explanation which does not involve right-handed neutrinos or even grand unification. In fact, it could be that the apparent merging of the running $SU(3) \times SU(2) \times U(1)$ gauge coupling constants at high energies ($E \sim 10^{15}$ GeV) would not signal the appearance of a unified gauge group such as $SO(10)$ but the onset of the nonperturbative dynamics responsible for the compositeness of the gauge bosons [30, 31].³ A further consequence of this nonperturbative dynamics might be the appearance of an effective coupling term (2b) which violates $B - L$, in addition to the $B + L$ violation inherent to the electroweak chiral gauge theory [15–17]. In this way, the conservation of both baryon number $B = (B + L)/2 + (B - L)/2$ and lepton number $L = (B + L)/2 - (B - L)/2$ would be only approximate at low energies, because the fundamental fermionic constituents would not care about these quantum numbers. The $SU(3) \times SU(2) \times U(1)$ gauge symmetry would be an emergent symmetry and the small neutrino mass scale would be a remnant of this state of affairs.

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large parentheses (these derivatives are to be contracted with condensate vectors or tensors).

³ Without the simple gauge group of grand unification, there would be no proton decay and no magnetic-monopole soliton (for original references and brief reviews on both topics, see, e.g., Ref. [12]).

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